symmetry requirements to a neighborhood of the superconvergent points, with general meshes allowed far enough away from the points. The case of superconvergent behaviour of the $L^2$ projection is studied as well, since the techniques are similar to those used to study the finite element solution but analysis for the $L^2$ projection is somewhat simpler.

Other topics in the book include superconvergence by "trivial, or not so trivial" postprocesses, for example, the use of difference quotients on translation invariant meshes as superconvergent approximations of derivatives and the use of various averaging operators to achieve superconvergence; extensions to nonlinear problems; extensions to meshes which are smooth mappings of meshes with sufficient invariance properties; superconvergence results for boundary integral operators on one-dimensional boundary curves; and numerical studies to locate superconvergent points.

Superconvergence is presented in these notes as a fascinating and challenging area of mathematical analysis—the question of its significance in practical computation is not a major concern. The treatment is extensive, the references nearly exhaustive, the proofs complete, and the exposition clear and precise. The book will surely be appreciated by researchers wanting some guidance through the vast literature on superconvergence, and by readers who appreciate the intricate and subtle craft of technical finite element numerical analysis.

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**2[35R30]**—*Inverse problems in diffusion processes*, Heinz W. Engl and William Rundell (Editors), SIAM Proceedings Series, SIAM, Philadelphia, PA and GAMM, Regensburg, Germany, 1995, xii+232 pp., $25\frac{1}{2}$ cm, softcover, $58.00

The eleven papers in this book are based on some of the invited talks at a conference held in Austria during the summer of 1994.

Three papers deal with the inverse heat conduction problem: one by Beck on the function specification method, one by Eldén on a numerical method using Tikhonov regularization, and one by Murio, Liu, and Zheng on a mollification method for stabilizing the inverse problem.

Two papers deal with theoretical aspects of regularization: Seidman with general considerations in dealing with ill-posed problems, and Chavent with recent results on the regularization of nonlinear least squares problems.

Three papers deal with the determination of unknown coefficients in second-order parabolic equations. In particular, Isakov considers identifiability from lateral and final data; Lowe and Rundell consider identifiability using boundary fluxes from interior sources; and it first reviews the literature and then discusses methods based on transforming the inverse problem to one involving a nonlocal functional.

The paper by Kunisch is a survey of some recent work on numerical methods for estimating the coefficients of elliptic equations.

The paper by Vainikko uses projection discretization schemes with Tikhonov regularization to deal with an inverse problem in groundwater filtration.

The paper by Lorenzi deals with finding the characteristics of one-dimensional dispersive media governed by Maxwell's equations.

The book is a valuable guide to the current state of knowledge about inverse problems in diffusion processes.

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**3[65-02, 45A55, 65D32, 65D30]**—*Lattice methods for multiple integration*, by I. H. Sloan and S. Joe, Oxford University Press, New York, 1994, xii+239 pp., 24 cm, $69.95

When Milton Abramowitz said in his article [1] that a common experience of applied mathematicians was to have a scientist come to the office and say "I have an integral", he was primarily referring to one-dimensional integrals. In the early days of numerical computing, multiple-dimensional integrals (cubatures) were avoided if possible, or manipulated into one-dimensional problems. As computing power increased, however, more attention was directed to the numerical evaluation of practical integrals in several variables. But long before numerical libraries contained reliable cubature methods, statisticians and others were computing high-dimensional integrals using Monte Carlo techniques, and mathematicians were producing elegant numerical approximations, based on rules for integrating polynomials or trigonometric polynomials exactly. (For an interesting history of cubatures, from Maxwell's brick to adaptive techniques, see [2].) Lattice methods provide a link between elegant, if impractical, cubature methods, and the practical, if inaccurate, Monte Carlo methods of the statisticians. They are of interest in themselves, regardless of applications, and are the basis of an algorithm for integration over hypercubes of, in theory, any dimension. The preface of this book states that it is aimed not only at those who might be interested in lattice methods for their own sake, but also those who have practical integrals to compute. The book certainly fulfills the first claim, but it is not clear that it provides practical help to those with high-dimensional integrals to approximate.

Chapter 1 provides a nice introduction to cubature, and to the idea of trigonometric degree. The main topic of the book is introduced in Chapter 2, where lattice rules are defined, and the early history of these is discussed. (The history of lattice rules is scattered throughout the book, with recent results and historical summaries appearing in several chapters.) In Chapter 3 the concept of *rank* of a lattice rule is introduced, together with a canonical form for a general lattice rule, based on *invariants* of the rule. The original lattice rules, called *rank*-1 rules, are discussed briefly in Chapter 4, and higher rank rules are dealt with in Chapters 5–7. Since lattice rules are intended for periodic integrands, some modifications are required either to the function or to the formula if the function is not periodic. Methods for periodising the function are discussed in Chapter 2, while modifications of the rules are discussed in Chapter 8. Chapter 9 contains some miscellaneous topics which do not fit anywhere else in the scheme of the book. This takes us to page 164 in a book which, apart from the appendices, contains 215 pages. In the remaining 50 or so pages, practical methods for integration, and comparisons with existing methods,